Digital Signal Processing

POSTECH
Department of Electrical Engineering
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Class information

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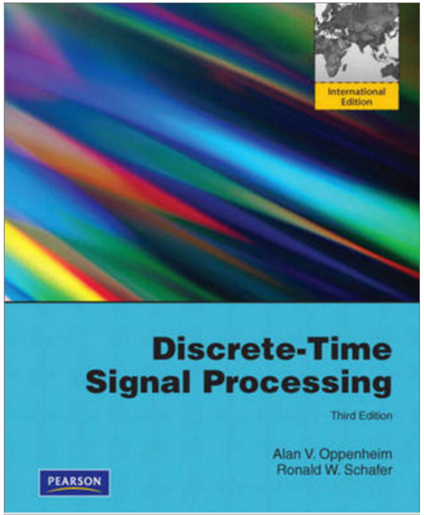
Class information

- Course materials
  - Go to https://www.icl.postech.ac.kr/
  - Sign in with your POSTECH email account
  - Go to Classes-EECE 451
  - Download lecture slides
  - Will post homework as well

Class information

- Textbook: Discrete-Time Signal Processing 3rd international ed.

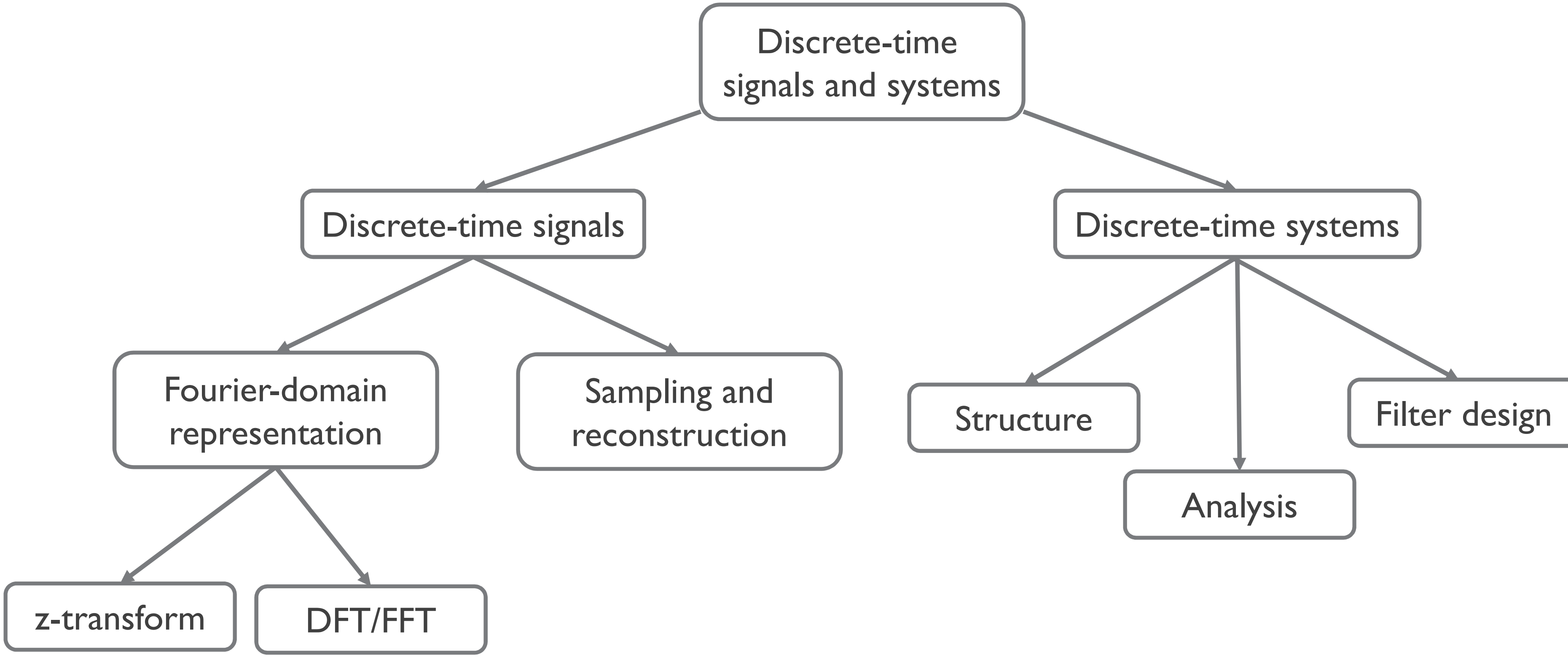


- Grade percentages
  - A 30%
  - B 50%
  - C 20%
  - F is possible if
    - the total score is below 10 out of 100
    - you miss half of the regular courses (i.e., 10)
  - Every 5 absents results in grade degradation

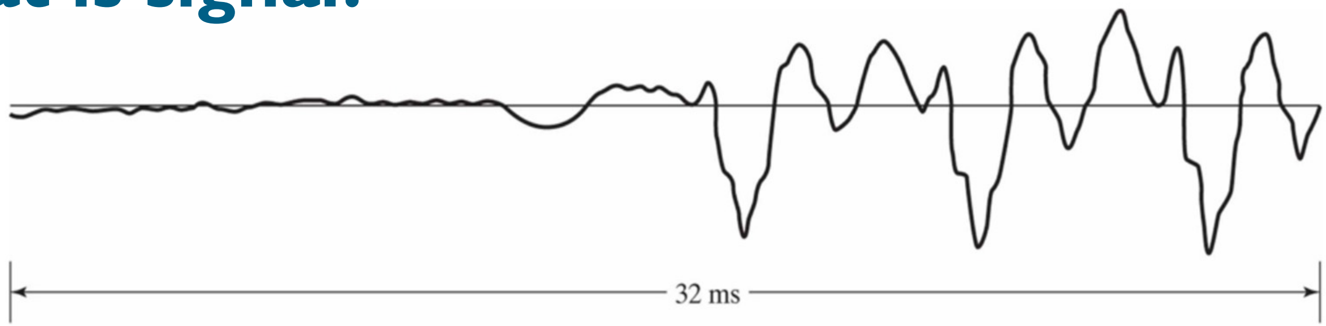
Class information

- Evaluation
  - MATLAB homework 15%
    - Do not copy but you can get help from colleagues
      - **You should indicate on the hardcopy from whom you get help**
      - **Full credit will be 7 out of 10**
      - **Please! Be honest!!!**
    - Textbook homework will not be graded
  - Quiz 15%
    - Mostly the same problems from the textbook homework
    - Around 5 quizzes
    - Each quiz may have about 3 questions and last 30 minutes
  - Midterm 30%
  - Final exam 40%

Course at glance



What is signal?



- Flow of information

- Mathematically, function of one or more independent variables
  - Time: speech
  - Position: image

- Commonly, call independent variable as time
  - Discrete-time signal processing techniques can be applied to all variables!

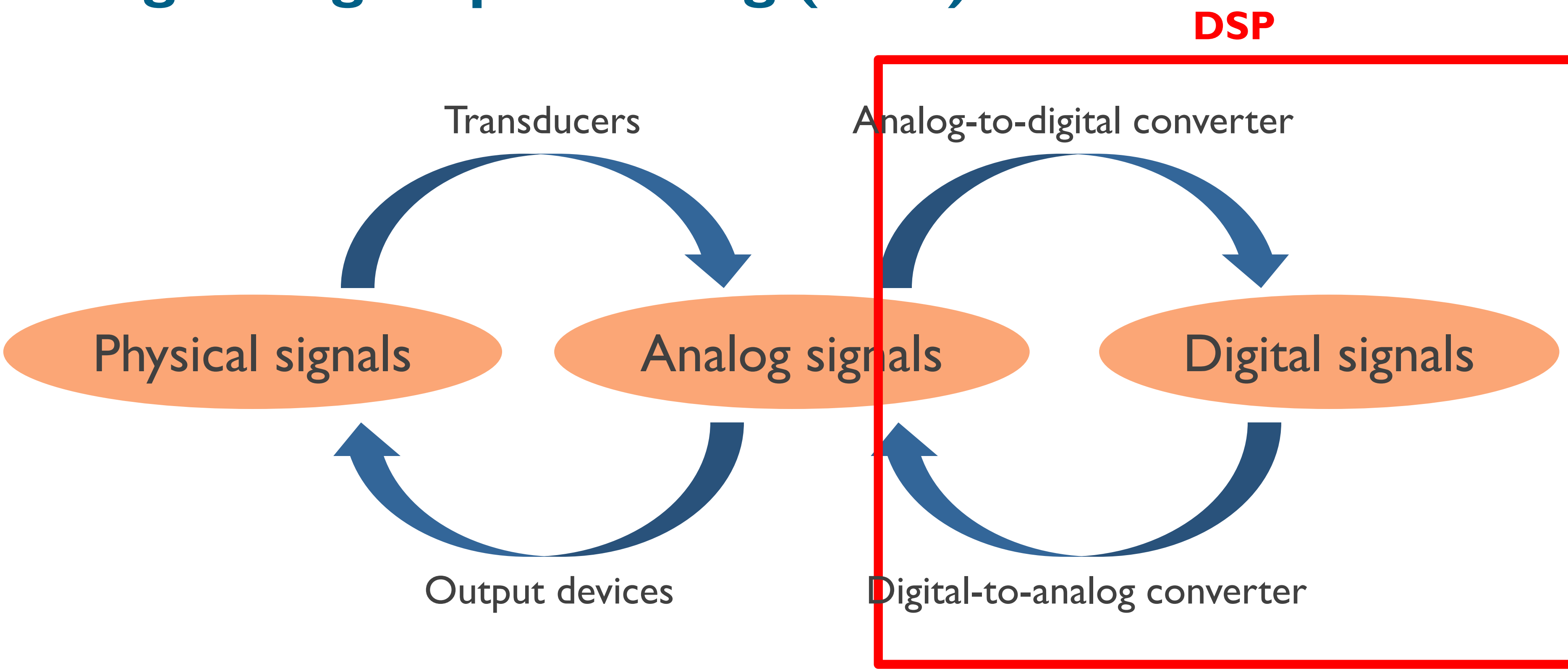
Signals in practice

- All physical signals are basically analog (continuous, infinite precision)
  - Sound
  - Picture (not digital images such as JPEG)
  - Smell
  - Texture

- Need to process signals for various purposes
  - Blur image
  - Increase bass sound in music

- Then, why digital processing, not analog?

Digital signal processing (DSP)



- Representation, transformation and manipulation of digital signals and the information the signals contain

Analog signal processing vs. digital signal processing

- Flexibility
  - Digital processing >>>>>>>>>>>>>>>>>>>> Analog processing
  - Analog circuit is usually designed to perform a specific task
- Complexity
  - Digital circuit <<<< Analog circuit but processing may be similar
- Speed
  - Digital processing < Analog processing
  - Not a problem due to extremely powerful processors
- Precision
  - Digital processing < Analog processing
  - Digital processing also can have very high resolution

DSP applications

- Communications
- Radar
- Image processing
- Speech processing
- Data storage
- Medical imaging
- Control
- Financial engineering
- And so much more!

DSP History

- Prior to 1950's: analog signal processing
  - Using electronic circuits or mechanical devices
- 1950's-1960's: computer simulation before analog implementation
  - Non-real time
- 1965: Fast Fourier Transforms (FFTs) by Cooley and Tukey
  - Make real time DSP possible
- 1980-present: IC technology boosting DSP

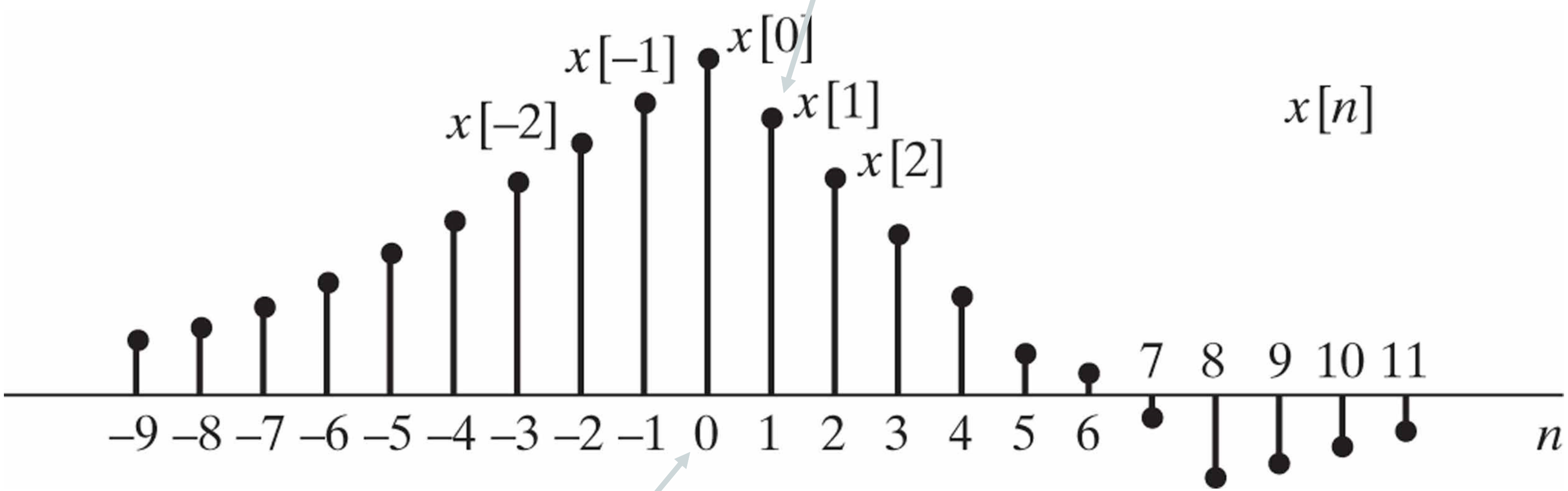
Digital signal processing demos

- Sound processing
- Image processing on pixels
- Image processing in frequency domain

Discrete-Time Signals

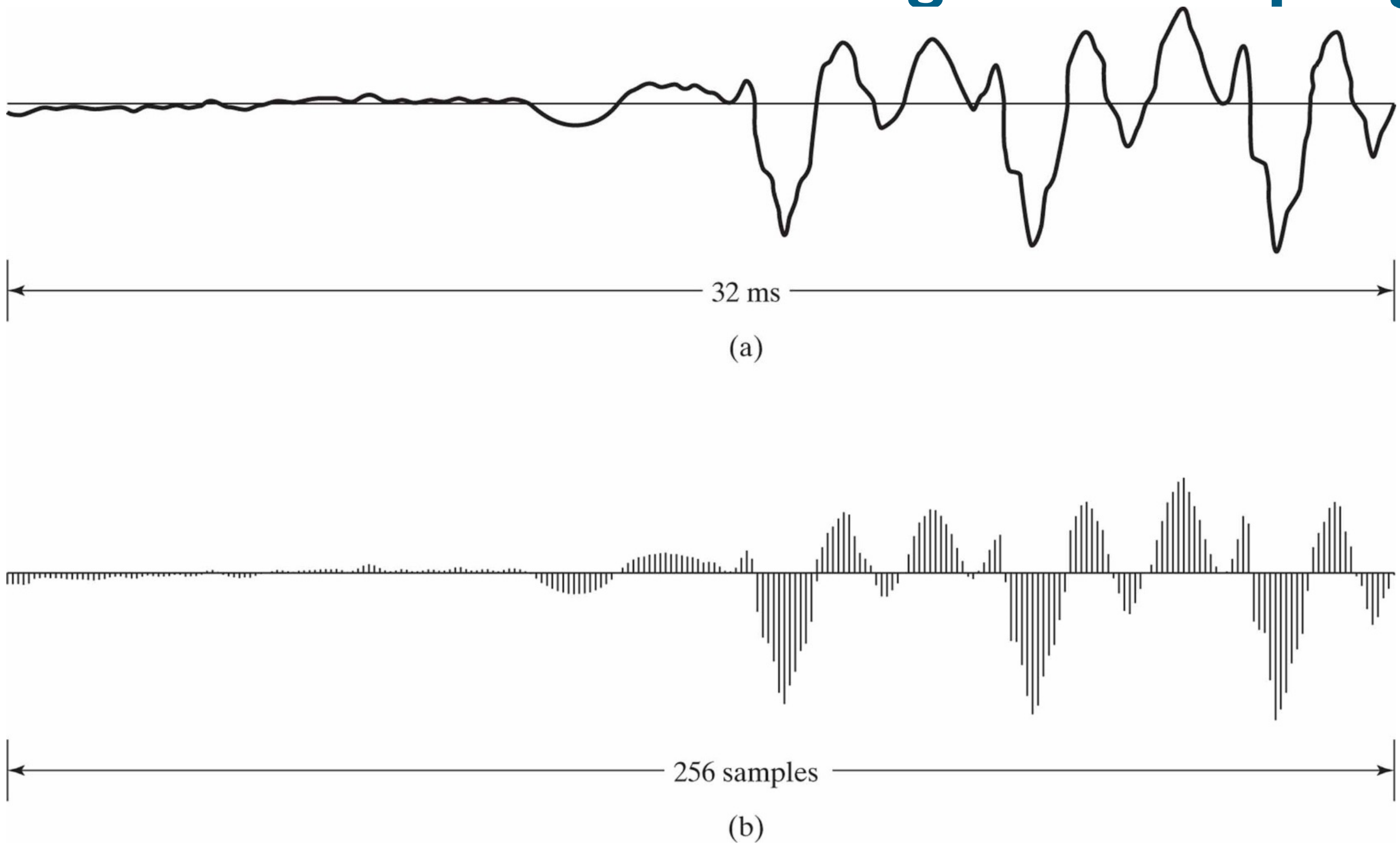
Discrete-time signals

Infinite precision: discrete-time signals
Finite precision: digital signals



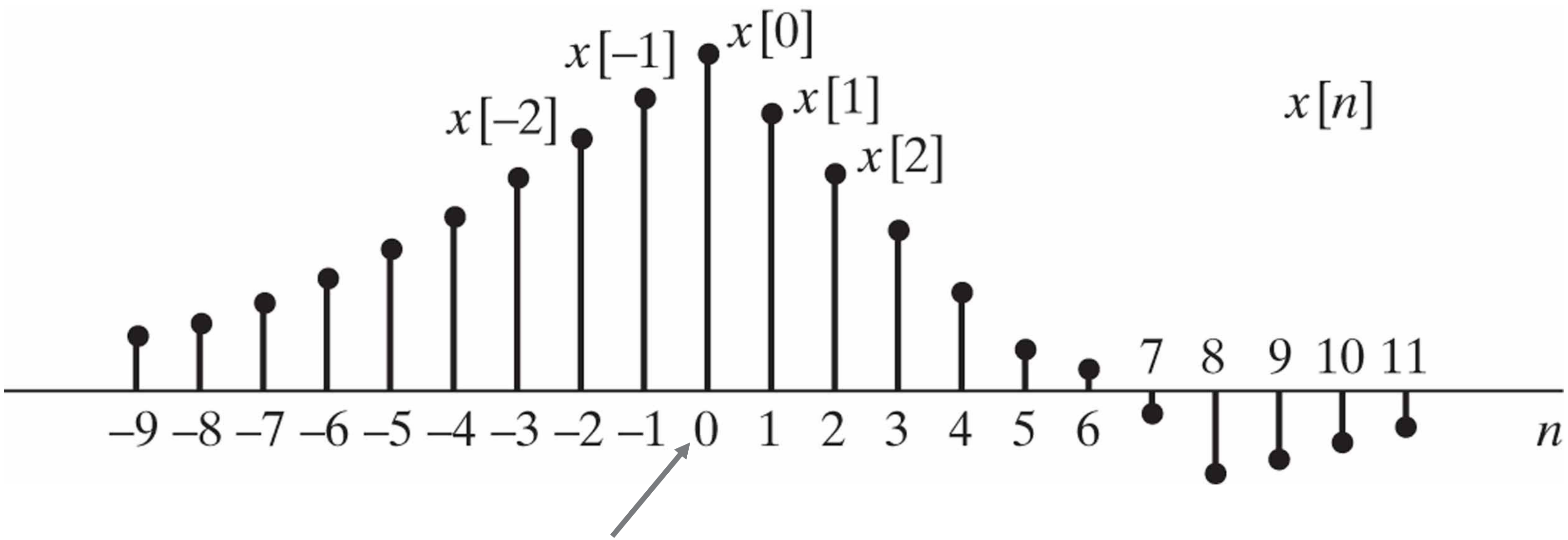
Indexed by integers (treat as time)

How to obtain discrete-time signals? Sampling!



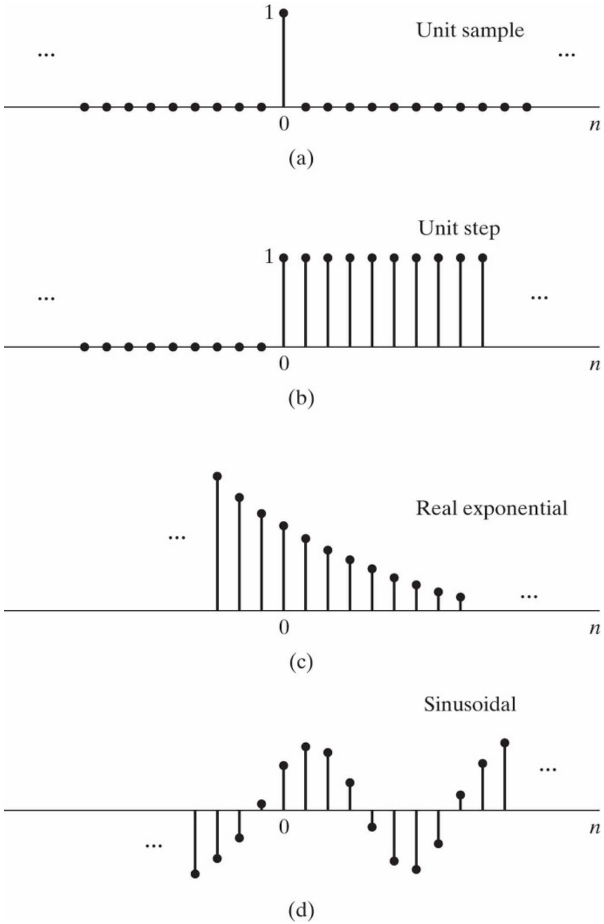
What is the sampling rate of this signal?

Do not get confused!

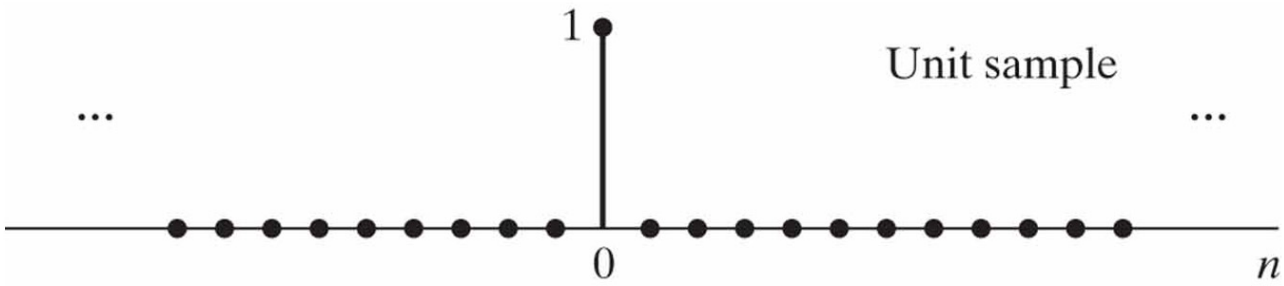


Values only defined in integer indices, i.e., $x[0.5] \neq 0$.
$x[0.5]$ is simply not defined at all.

Basic discrete-time sequences

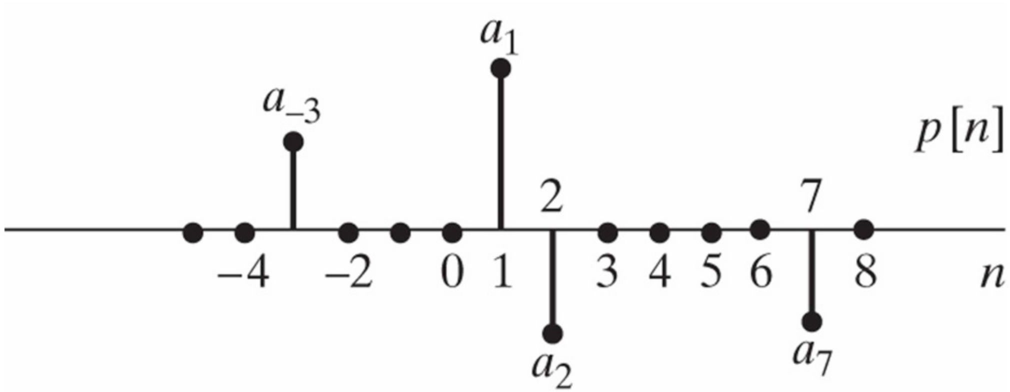


Unit sample sequence



- The role similar to the unit impulse function in continuous-time signals
- $\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$
- Often called discrete-time impulse or simply impulse

Important role of unit sample sequence



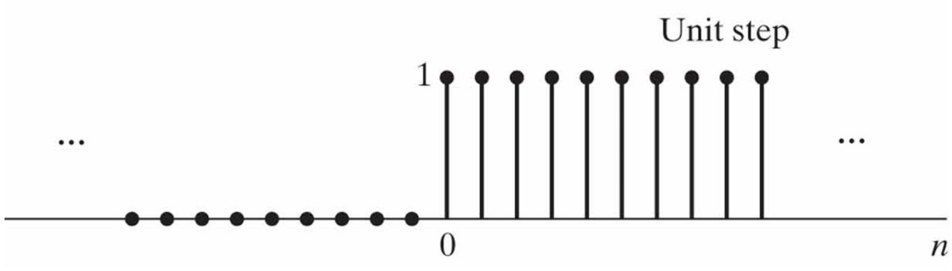
$$p[n] = a_{-3}\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-2] + a_7\delta[n-7]$$

- Any arbitrary sequence can be represented as unit sample sequence

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Memorize this form!

Unit step sequence

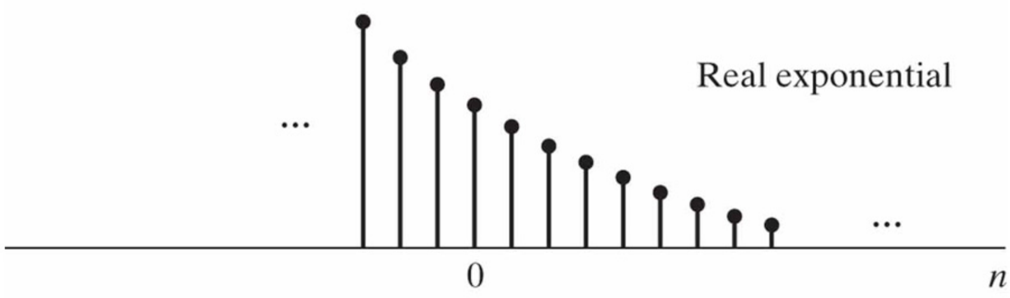


- $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} = \sum_{k=-\infty}^{n} \delta[k]$ → Why is this correct?
- Another interpretation of unit step sequence

$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots$$
$$= \sum_{k=0}^{\infty} \delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

Exponential sequences



$0 < \alpha < 1$

A positive

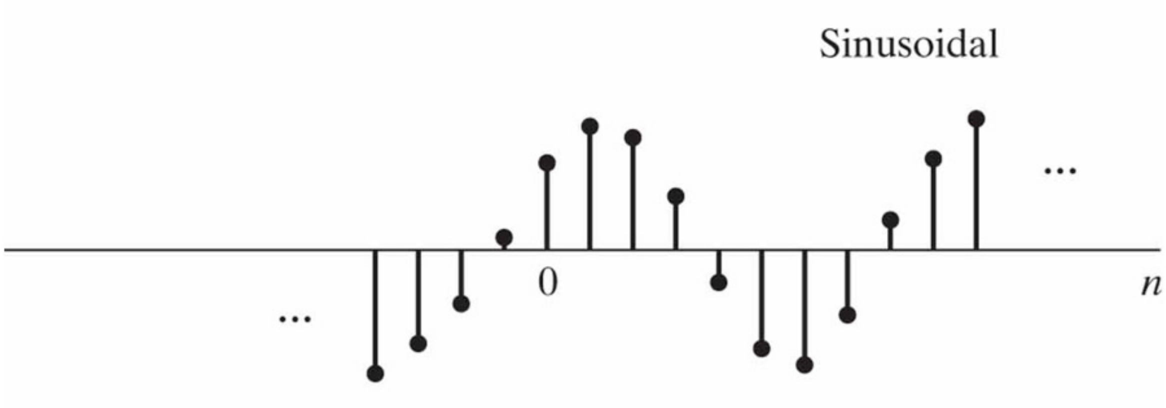
- General form: $x[n] = A\alpha^n$
- If *A* and alpha are real, x[n] is real. In general, they are complex.

$$\alpha = |\alpha| e^{j\omega_0}$$
$$A = |A| e^{j\phi}$$

$$x[n] = A\alpha^n = |A|e^{j\phi}|\alpha|^n e^{j\omega_0 n}$$
$$= |A||\alpha|^n e^{j(\omega_0 n + \phi)}$$
$$= |A||\alpha|^n \cos(\omega_0 n + \phi) + j|A||\alpha|^n \sin(\omega_0 n + \phi)$$

- What will x[n] look like with $|\alpha| > 1,\ |\alpha| < 1,\ |\alpha| = 1$?

Sinusoidal sequences



- With $|\alpha| = 1$

$$x[n] = |A|e^{j(\omega_0 n + \phi)} = |A|\cos(\omega_0 n + \phi) + j|A|\sin(\omega_0 n + \phi)$$

Frequency — Phase

Combine basic sequences

- An exponential sequence that is 0 for n<0

$$x[n] = \begin{cases} A\alpha^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$
$$= A\alpha^n u[n]$$

Properties of exponential and sinusoidal sequences

- Because n is always integer, with arbitrary integer r,

$$x[n] = Ae^{j(\omega_0+2\pi r)n} = Ae^{j\omega_0 n}e^{j2\pi rn} = Ae^{j\omega_0 n}$$

$$x[n] = A\cos[(\omega_0 + 2\pi r)n + \phi] = A\cos(\omega_0 n + \phi)$$

➡ Complex exponential and sinusoidal sequences with frequencies $(\omega_0 + 2\pi r)$, where r is an integer, are indistinguishable from one another.

- Periodicity

Period

$$x[n] = x[n+N], \quad \text{for all } n$$

  - For exponential sequence to be periodic, i.e., $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$ for all n, it is necessary and sufficient to have $\omega_0 N = 2\pi k$

➡ $e^{j\omega_0 n}$ may not be periodic depending on frequency ω_0

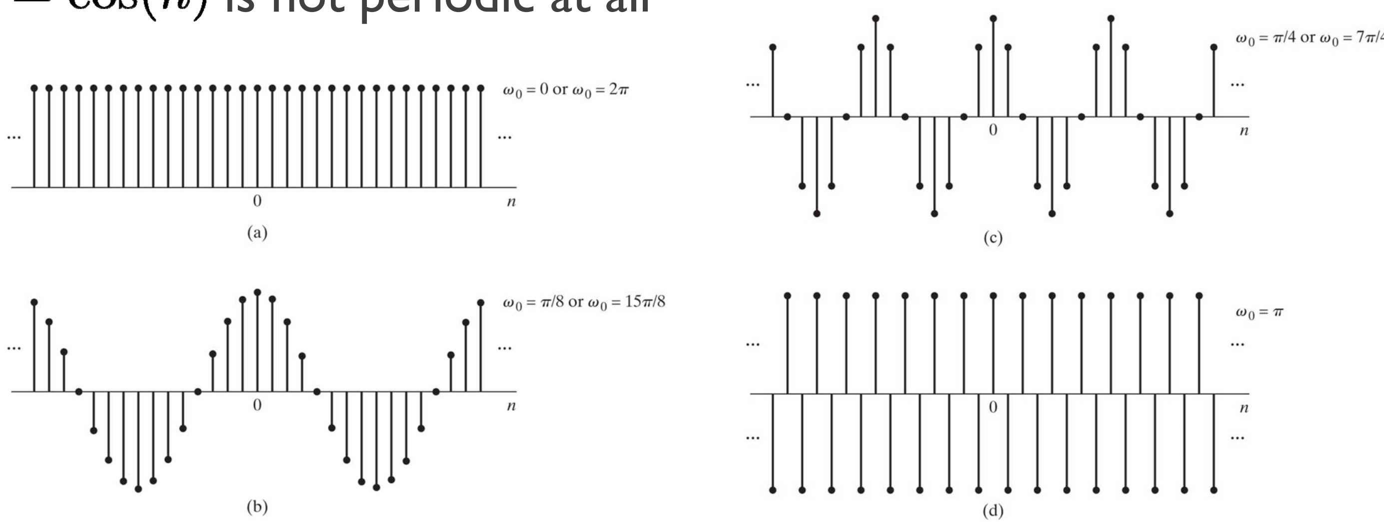
Periodicity examples

- $x[n] = \cos(\pi n/4)$ is periodic with period N=8
- $x[n] = \cos(3\pi n/8)$ is periodic with period N=16

} Different from continuous-time sinusoidal signals

- $x[n] = \cos(n)$ is not periodic at all

$\cos(\omega_0 n)$



Number of distinguishable frequencies

- Let $\omega_k = 2\pi k/N$

$$\omega_0 = 0, \; \omega_1 = 2\pi/N, \cdots, \; \omega_{N-1} = 2\pi(N-1)/N, \; \omega_N = 2\pi$$

Same frequency in exponential sequences

- There are N distinguishable frequencies that are periodic with period N

➡ Basic for discrete-time Fourier analysis later

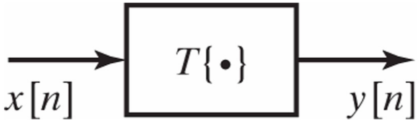
Discrete-Time Systems

Definition of discrete-time system

- Transformation or operator that maps an input x[n] into an output y[n]

$$y[n] = T\{x[n]\}$$

Mathematical definition



Pictorial definition

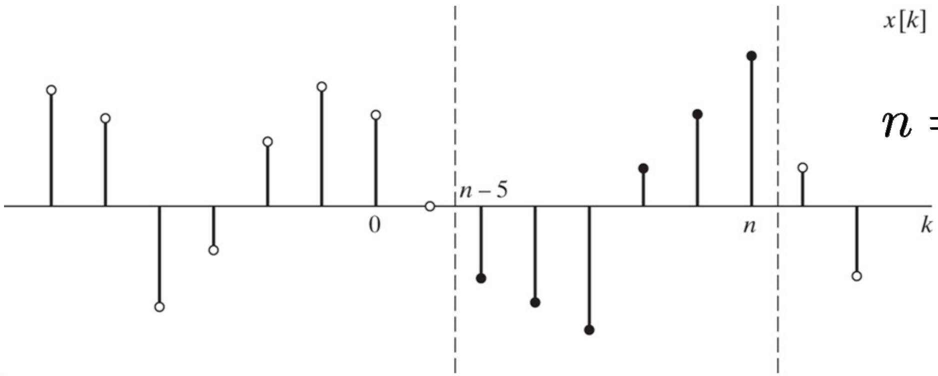
Examples of discrete-time systems

- The ideal delay system

$$y[n] = x[n - n_d], \quad -\infty < n < \infty$$

- Moving average

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$$



$$n = 7, \; M_1 = 0, \; M_2 = 5$$

Important properties of systems

- Memoryless systems
- Linear systems
- Time-invariant systems
- Causality
- Stability

Memoryless systems

- The output of y[n] at every value of n depends only on the input x[n] at the same value of n

- Example: $y[n] = (x[n])^2, \quad \text{for all } n$

- Examples of systems with memory
  - Ideal delay $y[n] = x[n - n_d], \quad -\infty < n < \infty$
  - Moving average $y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$

Linear systems

- A system is linear if and only if

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$$

Additivity property

and

$$T\{ax[n]\} = aT\{x[n]\} = ay[n]$$

Scaling property (homogeneity)

where a is an arbitrary constant.

- Combined into superposition

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\} = ay_1[n] + by_2[n]$$

for arbitrary constants a and b.

Examples of linear systems

- Ideal delay $y[n] = x[n - n_d], \quad -\infty < n < \infty$
- Moving average $y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$
- What about squaring system $y[n] = (x[n])^2, \quad \text{for all } n$?
- What about accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$?

Time-invariant systems

- Time shift of the input x[n] causes a corresponding shift in the output y[n]
- Let $y[n] = T\{x[n]\}$

  If $x_1[n] = x[n - n_0]$, then $y_1[n] = T\{x_1[n]\} = y[n - n_0]$ for all n_0

- Example:
  - Accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$ is time-invariant.

    Proof: Compare $y_1[n] = T\{x[n - n_d]\}$ and $y[n - n_0]$

Causality

- For every choice of n_0, the output sequence value at the index $n = n_0$ depends only on the input sequence values for $n \leq n_0$

- Current output is a function of only past and present inputs, not future inputs.

- Examples
  - Backward difference system is causal

$$y[n] = x[n] - x[n-1]$$

  - Forward difference system is non-causal

$$y[n] = x[n+1] - x[n]$$

Stability

- Many different definitions of 'stability' exist
- We focus on bounded-input bounded-output (BIBO) stability

- A system is BIBO stable iff every bounded input sequence produces a bounded output sequence.

- Input is bounded if there exists a fixed positive finite value B_x such that

$$|x[n]| \leq B_x < \infty, \quad \text{for all } n$$

With bounded input, there exists a fixed positive finite values B_y such that

$$|y[n]| \leq B_y < \infty, \quad \text{for all } n$$

Stability examples

- Squaring system $y[n] = (x[n])^2, \quad \text{for all } n$ is BIBO stable

- Accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$ is not BIBO stable

  Proof: Check with the input $x[n] = u[n]$

Linear time-invariant (LTI) systems

- LTI systems have significant signal-processing applications
- Recall that the input sequence can be represented as

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Will call this impulse response

- Let $h[n] = T\{\delta[n]\}$
- Output becomes $y[n] = T\{x[n]\} = T\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\}$

Work as constants

$$= \sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\}$$

Linear

$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Time-invariant

Discrete-time convolution

- Define the convolution sum operator

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= x[n] * h[n]$$

- Notational ambiguity

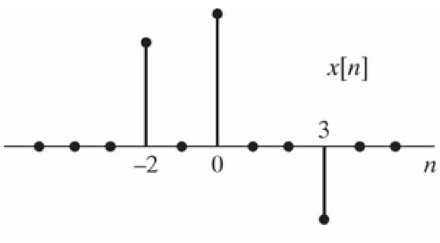
$$y[n-n_0] = \sum_{k=-\infty}^{\infty} x[k]h[n-n_0-k]$$

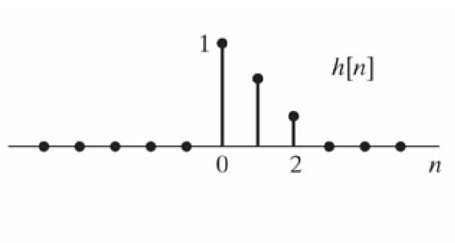
$$= x[n] * h[n-n_0]$$

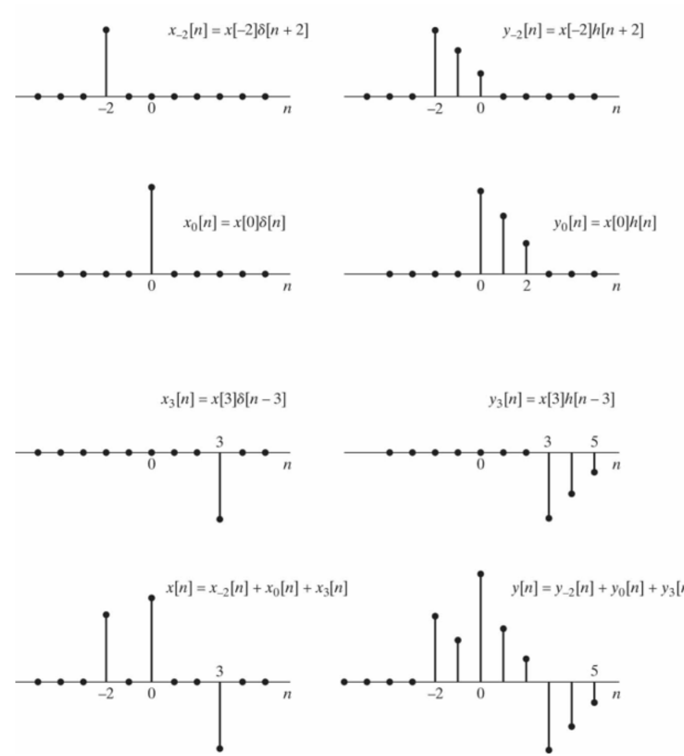
$$\neq x[n-n_0] * h[n-n_0] = y[n-2n_0]$$

Computing discrete-time convolution

- There are several ways to compute discrete-time convolution
- 1) Superposition of responses to individual samples of the input

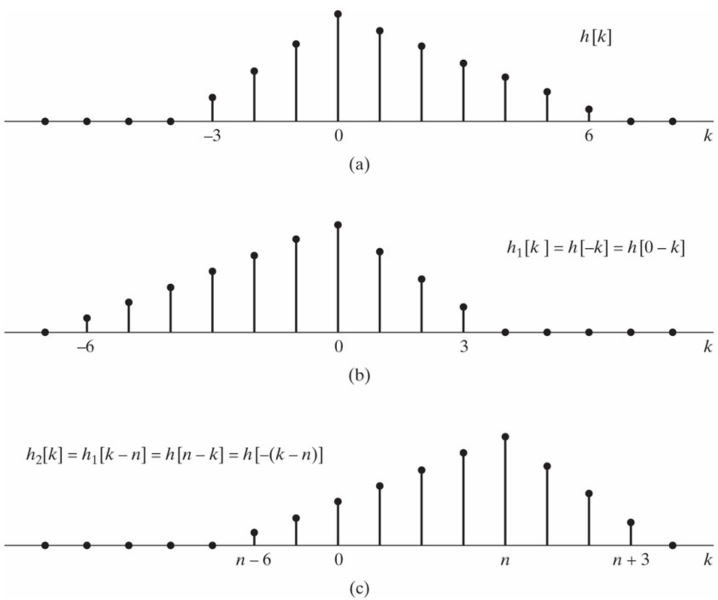






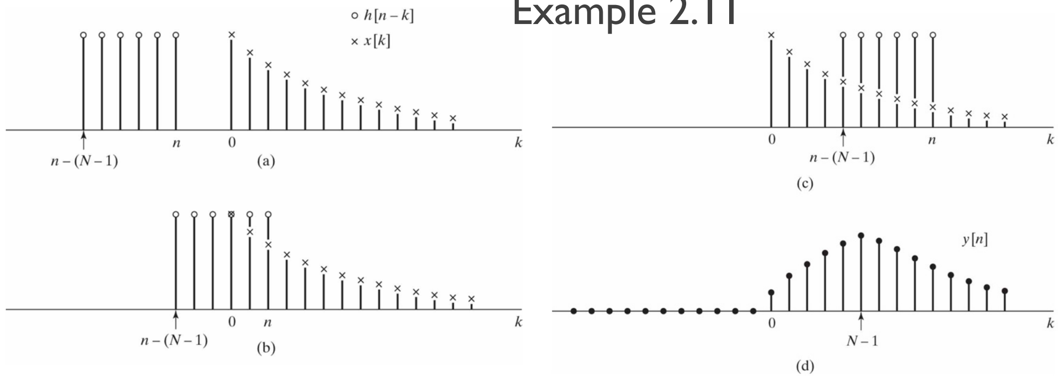
Computing discrete-time convolution

- 2) More systematical approach
  - Exploit h[n-k]



- Step 1: reverse h[k]
- Step 2: delay h[k] by n samples, i.e., h[n-k]
- Step 3: For each output sample y[n], multiply x[k] and h[n-k], then sum the whole products

Example 2.11



$$x[n] = a^n u[n], \;\; h[n] = u[n] - u[n-N]$$